Programming paradigms 1

Detailed evaluation of the 2nd exam

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Outline

1. ((if 0 + -) 1 (+ 1 1))
2. (define (x) (if x 0 1))
3. (+ (if (< 0 1) 0 1) 2)
4. (+ (define x 0) 1)
5. ((+) 1 2)

((if 0 + -) 1 (+ 1 1))

$\mathsf{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$

((if 0 + -) 1 (+ 1 1))

$$\mathsf{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\mathsf{Eval}[(\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

((if 0 + -) 1 (+ 1 1))

$$\text{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[(\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$$

((if 0 + -) 1 (+ 1 1))

$$\text{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[(\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$$

$$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, \texttt{0}, \texttt{+}, \texttt{-}] = \ldots$$

((if 0 + -) 1 (+ 1 1))

$$\text{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[(\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$$

$$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, \texttt{0}, \texttt{+}, \texttt{-}] = \ldots$$

$$\text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, \texttt{0}, \texttt{+}, \texttt{-}] = \ldots$

$\qquad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\qquad \texttt{0} \neq \text{'\#f'}$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-})\ \texttt{1}\ (\texttt{+}\ \texttt{1}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if}\ \texttt{0}\ \texttt{+}\ \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, \texttt{0}, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\quad\quad \texttt{0} \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 + \texttt{-})\ 1\ (\texttt{+}\ 1\ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 + \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 \texttt{ + -}) \ 1 \ (\texttt{+ } 1 \ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 \texttt{ + -}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Eval}[(\texttt{+ } 1 \ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 \texttt{ + -}) \ 1 \ (\texttt{+ } 1 \ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 \texttt{ + -}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Eval}[(\texttt{+ } 1 \ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 + \texttt{-})\ 1\ (\texttt{+}\ 1\ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 + \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Eval}[(\texttt{+}\ 1\ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 \texttt{ + -}) \ 1 \ (\texttt{+ } 1 \ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 \texttt{ + -}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Eval}[(\texttt{+ } 1 \ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 + \texttt{-})\ 1\ (\texttt{+}\ 1\ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 + \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, \texttt{+}, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Eval}[(\texttt{+}\ 1\ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Apply}[\text{'pr. proc. of sum.'},\ 1,\ 1] = 2$

((if 0 + -) 1 (+ 1 1))

$\text{Eval}[((\texttt{if } 0 + \texttt{-})\ 1\ (+\ 1\ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[(\texttt{if } 0 + \texttt{-}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, 0, +, \texttt{-}] = \ldots$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad 0 \neq \text{'\#f'}$

$\quad\quad \text{Eval}[+, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Eval}[(+\ 1\ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[+, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad \text{Apply}[\text{'pr. proc. of sum.'},\ 1,\ 1] = 2$

$\text{Apply}[\text{'pr. proc. of sum.'},\ 1,\ 2] = 3$

Outline

1. ((if 0 + -) 1 (+ 1 1))
2. (define (x) (if x 0 1))
3. (+ (if (< 0 1) 0 1) 2)
4. (+ (define x 0) 1)
5. ((+) 1 2)

(define (x) (if x 0 1))

$$\text{Eval}[(\texttt{define}\ (\texttt{x})\ (\texttt{if}\ \texttt{x}\ \texttt{0}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$$

(define (x) (if x 0 1))

$$\text{Eval}[(\texttt{define}\ (\texttt{x})\ (\texttt{if}\ \texttt{x}\ \texttt{0}\ \texttt{1})), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] = \text{'special form define'}$$

(define (x) (if x 0 1))

$$\text{Eval}[(\texttt{define}\ (\texttt{x})\ (\texttt{if}\ \texttt{x}\ 0\ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] = \text{'special form define'}$$

$$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, (\texttt{x}), (\texttt{if}\ \texttt{x}\ 0\ 1)] = \ldots$$

(define (x) (if x 0 1))

$\text{Eval}[(\texttt{define}\ (\texttt{x})\ (\texttt{if}\ \texttt{x}\ 0\ 1)), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] = \text{'special form define'}$

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, (\texttt{x}), (\texttt{if}\ \texttt{x}\ 0\ 1)] = \ldots$

$\quad$ **Error:** define: '(x)' is not a symbol.

Outline

1. ((if 0 + -) 1 (+ 1 1))
2. (define (x) (if x 0 1))
3. (+ (if (< 0 1) 0 1) 2)
4. (+ (define x 0) 1)
5. ((+) 1 2)

(+ (if (< 0 1) 0 1) 2)

$\mathsf{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

(+ (if (< 0 1) 0 1) 2)

$$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

(+ (if (< 0 1) 0 1) 2)

$$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

$$\text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] =$ 'special form if'

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\qquad \text{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

(+ (if (< 0 1) 0 1) 2)

$\mathsf{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\mathsf{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] =$ 'special form if'

$\mathsf{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\mathsf{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. less than'

(+ (if (< 0 1) 0 1) 2)

$\mathsf{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\mathsf{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] =$ 'special form if'

$\mathsf{Apply}_{\mathcal{P}_{\mathrm{G}}}[$'special form if', $(\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\mathsf{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. less than'

$\mathsf{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\qquad \text{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad\quad \text{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. less than'}$

$\qquad\quad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\qquad\quad \text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\qquad \text{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad\quad \text{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. less than'}$

$\qquad\quad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\qquad\quad \text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

$\qquad \text{Apply}[\text{'pr. proc. less than'},\ \texttt{0},\ \texttt{1}] = \text{'\#t'}$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\qquad \text{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad\quad \text{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. less than'}$

$\qquad\quad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\qquad\quad \text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

$\qquad \text{Apply}[\text{'pr. proc. less than'},\ \texttt{0},\ \texttt{1}] = \text{'\#t'}$

$\qquad \text{'\#t'} \neq \text{'\#f'}$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\qquad \text{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\qquad\quad \text{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. less than'}$

$\qquad\quad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\qquad\quad \text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

$\qquad \text{Apply}[\text{'pr. proc. less than'},\ \texttt{0},\ \texttt{1}] = \text{'\#t'}$

$\qquad \text{'\#t'} \neq \text{'\#f'}$

$\qquad \text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1})\ \texttt{2}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\text{Eval}[(\texttt{if}\ (\texttt{<}\ \texttt{0}\ \texttt{1})\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ \texttt{0}\ \texttt{1}), \texttt{0}, \texttt{1}] = \ldots$

$\text{Eval}[(\texttt{<}\ \texttt{0}\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. less than'}$

$\text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

$\text{Apply}[\text{'pr. proc. less than'},\ \texttt{0},\ \texttt{1}] = \text{'\#t'}$

$\text{'\#t'} \neq \text{'\#f'}$

$\text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\text{Eval}[\texttt{2}, \mathcal{P}_{\mathrm{G}}] = \texttt{2}$

(+ (if (< 0 1) 0 1) 2)

$\text{Eval}[(\texttt{+}\ (\texttt{if}\ (\texttt{<}\ 0\ 1)\ 0\ 1)\ 2), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad \text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\quad \text{Eval}[(\texttt{if}\ (\texttt{<}\ 0\ 1)\ 0\ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad \text{Eval}[\texttt{if}, \mathcal{P}_{\mathrm{G}}] = \text{'special form if'}$

$\quad \text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form if'}, (\texttt{<}\ 0\ 1), 0, 1] = \ldots$

$\quad\quad \text{Eval}[(\texttt{<}\ 0\ 1), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\quad\quad\quad \text{Eval}[\texttt{<}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. less than'}$

$\quad\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad\quad\quad \text{Eval}[1, \mathcal{P}_{\mathrm{G}}] = 1$

$\quad\quad \text{Apply}[\text{'pr. proc. less than'},\ 0,\ 1] = \text{'\#t'}$

$\quad\quad \text{'\#t'} \neq \text{'\#f'}$

$\quad\quad \text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\quad \text{Eval}[2, \mathcal{P}_{\mathrm{G}}] = 2$

$\text{Apply}[\text{'pr. proc. of sum.'},\ 0,\ 2] = 2$

Outline

1. ((if 0 + -) 1 (+ 1 1))
2. (define (x) (if x 0 1))
3. (+ (if (< 0 1) 0 1) 2)
4. (+ (define x 0) 1)
5. ((+) 1 2)

(+ (define x 0) 1)

$\mathsf{Eval}[(\texttt{+}\ (\texttt{define}\ \texttt{x}\ \texttt{0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

(+ (define x 0) 1)

$$\text{Eval}[(\texttt{+}\ (\texttt{define}\ \texttt{x}\ \texttt{0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

(+ (define x 0) 1)

$$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

$$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$$

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

(+ (define x 0) 1)

$\mathsf{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$

$\mathsf{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\mathsf{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] = \text{'special form define'}$

$\mathsf{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, \texttt{0}] = \ldots$

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, 0] = \ldots$

✓ $\texttt{x}$ is a symbol.

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, 0] = \ldots$

✓ $\texttt{x}$ is a symbol.

$\text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, 0] = \ldots$

✓ $\texttt{x}$ is a symbol.

$\text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\texttt{x} \mapsto_{\mathcal{P}_{\mathrm{G}}} 0$

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, 0] = \ldots$

✓ $\texttt{x}$ is a symbol.

$\text{Eval}[0, \mathcal{P}_{\mathrm{G}}] = 0$

$\texttt{x} \mapsto_{\mathcal{P}_{\mathrm{G}}} 0$

$=$ 'undefined'

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, \texttt{0}] = \ldots$

✓ $\texttt{x}$ is a symbol.

$\text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\texttt{x} \mapsto_{\mathcal{P}_{\mathrm{G}}} \texttt{0}$

$=$ 'undefined'

$\text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, \texttt{0}] = \ldots$

✓ $\texttt{x}$ is a symbol.

$\text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\texttt{x} \mapsto_{\mathcal{P}_{\mathrm{G}}} \texttt{0}$

$=$ 'undefined'

$\text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

Apply['pr. proc. of sum.', 'undefined', 1] =

(+ (define x 0) 1)

$\text{Eval}[(\texttt{+}\ (\texttt{define x 0})\ \texttt{1}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{+}, \mathcal{P}_{\mathrm{G}}] =$ 'pr. proc. of sum.'

$\text{Eval}[(\texttt{define x 0}), \mathcal{P}_{\mathrm{G}}] = \ldots$

$\text{Eval}[\texttt{define}, \mathcal{P}_{\mathrm{G}}] =$ 'special form define'

$\text{Apply}_{\mathcal{P}_{\mathrm{G}}}[\text{'special form define'}, \texttt{x}, \texttt{0}] = \ldots$

✓ $\texttt{x}$ is a symbol.

$\text{Eval}[\texttt{0}, \mathcal{P}_{\mathrm{G}}] = \texttt{0}$

$\texttt{x} \mapsto_{\mathcal{P}_{\mathrm{G}}} \texttt{0}$

$=$ 'undefined'

$\text{Eval}[\texttt{1}, \mathcal{P}_{\mathrm{G}}] = \texttt{1}$

Apply['pr. proc. of sum.', 'undefined', 1] =

Error: The procedure was applied with other arguments than numbers.

Outline

1. ((if 0 + -) 1 (+ 1 1))
2. (define (x) (if x 0 1))
3. (+ (if (< 0 1) 0 1) 2)
4. (+ (define x 0) 1)
5. ((+) 1 2)

((+) 1 2)

$\mathsf{Eval}[((+)\ 1\ 2), \mathcal{P}_{\mathrm{G}}] = \ldots$

((+) 1 2)

$$\mathsf{Eval}[((+)\ 1\ 2), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\mathsf{Eval}[(+), \mathcal{P}_{\mathrm{G}}] = \ldots$$

((+) 1 2)

$$\mathsf{Eval}[((+)\ 1\ 2), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\mathsf{Eval}[(+), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\mathsf{Eval}[+, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

((+) 1 2)

$$\text{Eval}[((+)\ 1\ 2), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[(+), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[+, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

$$\text{Apply}[\text{'pr. proc. of sum.'}] = 0$$

((+) 1 2)

$$\text{Eval}[((+)\ 1\ 2), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[(+), \mathcal{P}_{\mathrm{G}}] = \ldots$$

$$\text{Eval}[+, \mathcal{P}_{\mathrm{G}}] = \text{'pr. proc. of sum.'}$$

$$\text{Apply}[\text{'pr. proc. of sum.'}] = 0$$

Error: The first element did not evaluate to proc. or spec. form.